# On the Leaky Modes for Silver Nanowires on a Silica Substrate

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Abstract- An in-house developed full-vector finite-element imaginary-distance beam propagation method (FV-FE-IDBPM) is employed to calculate the modes of silver nanowires on a silica substrate. Leaky-mode solutions that have not yet been reported in the literature are presented to add knowledge to the current understanding about such plasmonic guiding structure. Complex effective indices are accurately solved from which the modal propagation lengths are determined.

### I. INTRODUCTION

Regarding circular-cylinder surface-plasmon-polariton (SPP) nanowires, although they have been appearing as sitting on a substrate in many experiments, guided-mode studies of such substrate-supported structures have not been widely conducted compared with the free-standing nanowires [1]. In [1], waveguiding properties of metal nanowires with dielectric substrates of different materials were investigated, in particular for the fundamental mode at 660-nm wavelength. Single-guiding-mode conditions versus the operating wavelength for silver nanowires on a silica substrate were investigated in [2] from visible to near-infrared up to 2000-nm wavelength, considering possible applications in high-capacity circuits for optical communication. In [2], circular silver nanowires immersed in a silica matrix were first solved for their first (fundamental) and second order guided modes, with cutoff wavelengths of the latter determined. But the second order modes were found not to play roles in the later analyzed substrate-supported cases. In this paper, we present our obtained leaky modes related to such second order guided modes.

The mode solutions in this work are based on an in-house developed full-vector finite-element imaginary-distance beam propagation method (FV-FE-IDBPM) [3], [4]. Ideas of getting the aforementioned leaky modes based on previous experience in other plasmonic waveguides are discussed in Section II. Section III shows the calculated effective refractive indices and propagation lengths for modes of the silica-supported silver nanowires of different radii. Section IV gives the conclusion.

## II. EXPERIENCE FROM OTHER PLASMONIC WAVEGUIDES

Consider the one-dimensional (1D) planar structure composed of air cover region, silver film, and silica substrate, as depicted in Fig. 1(a), whose modal characteristics have been useful in understanding those of the metal stripe plasmo-



Fig. 1. (a) Schematic of a planar air-silver-silica waveguide. (b) Crosssection of a metal stripe plasmonic waveguide. (c) Cross-section of a silica-substrate-supported silver nanowire.



Fig. 2.  $\text{Re}[H_x]$  profiles versus *y* position for (a) the pure guided mode and (b) the leaky mode of the planar structure of Fig. 1(a).

nic waveguide [5] with cross-section shown in Fig. 1(b). Taking the silver film thickness of 120 nm and the wavelength  $\lambda = 500$  nm, two transverse-magnetic (TM) modes are obtained for Fig. 1(a) using a planar-waveguide FE mode solver [5] with their transverse-magnetic-field ( $\operatorname{Re}[H_r]$ ) profiles versus y as shown in Fig. 2(a) and (b), respectively. In this paper, the complex permittivity for silver is adopted from [7], which is a curve-fitting version of the measured data in [8], as in [2]. The mode field in Fig. 2(a) which is mainly guided at the silver-silica interface represents a pure guided mode, while that in Fig. 2(b) is mainly at the air-silver interface with the leaky behavior in the substrate since its  $|n_{eff}|$  is smaller than the index of silica which is 1.45. The modal effective index,  $n_{eff}$ , is the complex modal propagation constant divided by the freespace wavenumber. These two modes are the counterparts of the leaky mode and the bound mode of the stripe waveguide of Fig. 1(b) and Ref. [5]. For the bound modes (not shown in this paper), the mode field is mainly distributed below the metal stripe in the substrate, resembling the characteristic of the profile in Fig. 2(a). There are leaky modes with the mode field mainly distributed above the metal stripe upper interface and field leakage into the substrate, resembling the characteristic of the profile in Fig. 2(b). We would thus expect the possibility of the existence of leaky modes for the silicasupported nanowire of Fig. 1(c), with the mode field mainly distributed above the top air-silver interface, as will be confirmed in the following.

# III. NUMERICAL RESULTS FOR THE NANOWIRES

For  $\lambda = 500$  nm and nanowire radius (*r*) of 160 nm, we have found the first and second order modes for the structure of Fig. 1(c), as shown in Figs. 3 and 4, respectively. The second order mode indeed is a leaky one possessing similar characteristics as those in the stripe waveguide of Fig. 1(b).





Fig. 4. Mode-field profiles of the second order mode of the structure of Fig. 1(c) when  $\lambda = 500$  nm and r = 160 nm. (a) Re[ $E_x$ ]. (b) Re[ $E_y$ ].



Fig. 5. Re[ $n_{eff}$ ] versus wavelength curves for the first order mode (cutoff at  $n_{eff} = 1.45$ ) and the second order mode (cutoff at  $n_{eff} = 1$ ) of the structure of Fig. 1(c) for three different nanowire radii.

Three different radii of the nanowire, r = 60 nm, 100 nm, and 160 nm, are then considered for showing Re[ $n_{eff}$ ] versus wavelength curves for the first and second order modes in Fig. 5, having cutoff  $n_{eff}$  of 1.45 and 1, respectively. The corresponding propagation-length results determined from Im[ $n_{eff}$ ] are shown in Fig. 6. The first-order-mode results agree with those given in [2]. The Re[ $n_{eff}$ ] versus wavelength curves for the second order mode of a nanowire in air without substrate for the same three different nanowire radii are plotted in Fig. 7. Note that the cutoff wavelengths are almost the same as those in Fig. 5

# IV. CONCLUSION

Waveguide modes of silver nanowires on a silica substrate have been calculated using an in-house developed FV-FE-IDBPM. Leaky-mode solutions with a similar field behavior as those in the metal stripe waveguide have been found. This kind of modes have not yet been reported in the literature.



Fig. 6. Propagation length versus wavelength curves corresponding to Fig. 5.



Fig. 7.  $\text{Re}[n_{eff}]$  versus wavelength curves for the second order mode of a nanowire in air for three different nanowire radii.

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