Advanced Control Schemes for Passively Mode-Locked Lasers: Coupled Lasers and Dual-Feedback Approaches

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Abstract—We investigate optical control schemes for passively mode-locked lasers (PMLLs). The coupling of two identical PMLLs provides a possibility to produce highly regular pulse trains by reducing the pulse timing jitter, superior to the single optical feedback scheme, while furthermore allowing for an effective doubling of the pulse repetition rate for proper coupling delay. A dual optical feedback scheme is identified as a way to tune the repetition rate of a single PMLL over several hundred MHz while maintaining very low timing jitter.

I. INTRODUCTION

Mode-locked lasers (MLLs) are widely used for the creation of high-frequency, ultra-short light pulses, e.g. for application in pump-probe experiments and optical data communication. Passively mode-locked lasers (PMLLs) have the advantage of easy fabrication and thus low cost, due to the absence of an external clock signal. The drawback of this simple setup, however, is a relatively large timing jitter, i.e., irregularity of the arrival times of subsequent pulses, due to statistical fluctuations in the MLL device. It has been previously shown [1], [2] that time-delayed optical feedback can greatly reduce their timing jitter. Here, we want to extend this approach to more advanced control schemes, by implementing an optical coupling of two PMLLs or a dual-feedback setup [3], as sketched in Fig. 1.

II. MODEL

We implement a DDE model describing a ring cavity PMLL, as introduced in Ref. [4]. This model was later extended to include optical feedback [5]. The set of three coupled delay differential equations describing the PMLL coupled to



Fig. 1. Sketch of the passively mode-locked laser (PMLL) control schemes: (a) PMLL with delayed optical feedback. (b) Delayed coupling of two PMLLs. G and Q denote gain and absorber sections, respectively.

 TABLE I

 PARAMETER VALUES USED IN NUMERICAL SIMULATIONS.

symbol	value	symbol	value
γ	$2.66 ps^{-1}$	r_s	25
γ_g	$1ns^{-1}$	κ	0.1
γ_q	$75ns^{-1}$		25ps
J_g	$0.12 ps^{-1}$	J_q	$0.3 ps^{-1}$

an external feedback cavity are

$$\dot{G}(t) = J_g - \gamma_g G(t) - e^{-Q(t)} \left(e^{G(t)} - 1 \right) |\mathcal{E}(t)|^2, \qquad (1)$$

$$\dot{Q}(t) = J_q - \gamma_q Q(t) - r_s e^{-Q(t)} \left(e^{Q(t)} - 1 \right) |\mathcal{E}(t)|^2, \quad (2)$$

$$\frac{1}{\gamma} \dot{\mathcal{E}}(t) = R(t-T)\mathcal{E}(t-T) - \mathcal{E}(t) + \beta \xi(t) + K_1^{\text{fb}} R(t-T-\tau_1)\mathcal{E}(t-T-\tau_1) + K_2^{\text{fb}} R(t-T-\tau_2)\mathcal{E}(t-T-\tau_2) + K_c R'(t-T-\tau_c)\mathcal{E}'(t-T-\tau_c)$$
(3)

The dynamical variables are the slowly varying electric field amplitude \mathcal{E} , the saturable gain G and the saturable loss Q. $R(t) \equiv \sqrt{\kappa}e^{\frac{1}{2}(G(t)-Q(t))}$ denotes the round-trip net gain. The parameters in the equations are: the cold cavity roundtrip time T, the full-width at half maximum γ of the Lorentzian-shaped filter function used to account for the finite width of the gain spectrum, the unsaturated gain J_g in the gain section, the unsaturated absorption J_q in the saturable absorber section, the carrier lifetimes in the gain and absorber sections γ_q^{-1} and γ_q^{-1} , the ratio of the saturation energies in the gain and absorber sections r_s , and the non-resonant losses are taken into account by κ . Spontaneous emission is included by a Gaussian white noise process $\xi(t)$ with the noise amplitude β . The optical feedback is described by the external cavity roundtrip times (delay times) $au_{1,2}$ with the corresponding feedback strengths $K_{1,2}^{\text{fb}}$. K_{c} and τ_c describe the coupling strength and delay time between two PMLLs, where the electric field $\mathcal{E}(t)$ couples to a second, identical set of equations (and vice versa), denoted by a prime.

III. RESULTS

We simulate the PMLL with parameters given in Tab.1. The solitary PMLL exhibits a pulse repetition rate around



Fig. 2. Long-term timing jitter in dependence on the optical delay times τ_1 for the PMLL with single optical feedback (blue), and on the coupling delay time τ_c for two coupled PMLLs. The delay times are given in units of the solitary PMLL inter-spike interval time $T_{\rm ISI}$. The dashed line denotes the timing jitter of the solitary PMLL. $K_1^{\rm fb} = 0.1$, $K_2^{\rm fb} = 0$, $K_c = 0.1$.

40GHz with a timing jitter of 5.7fs. Fig. 2 shows the PMLL performance in the presence of a single optical feedback line (blue, $K_1^{\rm fb}=0.1,~K_2^{\rm fb}=K_c=0$). It is known that the dynamics and timing jitter of the PMLL depend strongly on the optical feedback length, which enters Eq. (3) as the time delay τ_1 . Around the main resonances between feedback delay τ_1 and the pulse repetition interval $T_{ISI,0}$ the PMLL can be seen to exhibit regular mode-locking with a greatly reduced timing jitter. In between, irregular dynamics are observed, which can lead to an increase of the timing-jitter. In comparison, coupling two identical PMLLs yields an improvement over the singlefeedback timing-jitter reduction, reaching values as low as 0.4fs for the long-term timing jitter (red, $K_1^{\text{fb}} = K_2^{\text{fb}} = 0$, $K_c = 0.1$). Furthermore, also around second-order resonances, where the feedback delay is a half-integer multiple of the solitary PMLL repetition interval, a strong suppression of the timing jitter can be observed. Here, the two PMLLs operate in an alternating fashion, effectively doubling the repetition frequency. We therefore identify the coupling scheme as a reliable way to improve the timing jitter and increase the pulse repetition rate.

Now, we investigate the dual-feedback control scheme. We therefore look at a single PMLL by setting $K_1^{\rm fb} = K_2^{\rm fb} = 0.05$, and $K_c = 0$. Fig. 3a shows the long-term timing jitter in dependence of the two optical delays. A region of strong jitter suppression can be observed around the main resonance of both delays with the solitary laser pulse repetition interval. Furthermore, an accurate tuning of the two delays yields the possibility to change the repetition rate of the PMLL over several hundred MHz, while still maintaining the strong decrease in timing jitter. We identify an optimal parameter range which is shown by the dashed line in Fig. 3.

IV. CONCLUSION

We have investigated the possibility to optimize the operation of passively mode-locked lasers (PMLLs) by utilizing optical control schemes. We have found that the coupling of two PMLLs yields an improvement over the single optical feedback scheme in terms of timing jitter reduction. A proper choice of the coupling delay leads to an alternating operation



Fig. 3. (a) Long-term timing jitter for the single PMLL with a dual optical feedback scheme. The color code shows the timing jitter in dependence of the two feedback delays τ_1 , τ_2 . (b) Pulse repetition rate in dependence of the two feedback delays. The dashed line in both panels denotes a parameter range for which strong suppression of the timing jitter along with a tuning of the pulse repetition rate over several hundred MHz is possible. $K_1^{\rm fb} = K_2^{\rm fb} = 0.05$.

of the two PMLLs, effectively doubling the repetition rate. The coupling scheme therefore presents a promising technique to create high-repetition-frequency low-timing-jitter mode-locked pulses. Furthermore, we investigated the dual optical-feedback setup of a single PMLL. We found a possibility to tune its repetition rate over several hundred MHz while still maintaining very low timing jitter, thus allowing the emission of regular mode-locked pulses with a fixed repetition rate.

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